

自由节点样条函数平滑滤波技术

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摘 要: 考虑了目标跟踪和航天测控中测量数据的平滑问题. 提出了利用自由节点样条函数来改善多项式最小二乘滤波器设计的模型和方法. 用样条函数表示弹道, 能改善目标机动时用多项式表示弹道引入的截断误差较大的缺点, 提高弹道参数的估计精度. 仿真计算结果表明该方法能显著改善目标机动时的运动参数估计精度.

关键词: 自由节点样条函数; 平滑; 截断误差; 参数估计

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Data Smoothing and Filtering Technique by Free Node Spline Function

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Abstract: In this paper, we discussed the smoothing problem in target tracking and aerospace measurement. A novel model based on free node spline function is put forward to improve the design of the polynomial least square filter. Compared with a polynomial expression of trajectory, the spline expression of the trajectory can make the truncation error decrease greatly. Thus the trajectory parameter estimation accuracy is enhanced accordingly. Simulation results show that this method can improve the parameter estimation accuracy notably when target is maneuvering.

Key words: free node spline function; smoothing; truncation error; parameter estimation

1 引言

在目标跟踪中, 观测数据总含有随机误差, 必须通过平滑滤波处理, 才能得到真正反映目标飞行特性的位置、速度等参数. 虽然测量数据的随机误差特性非常复杂, 但在数据处理中, 常将其假设为高斯白噪声, 以简化数据处理过程^[1]. 在航天测量数据处理中, 由于精度要求高, 多采用最小二乘的多项式平滑、滤波及外推(以下统称为滤波)算法, 对仅含白噪声的测量值序列来说, 最小二乘算法也是最小方差算法. 文[1]给出了用偶次阶(二阶或四阶)中心平滑公式由位置观测数据微分求速的方法, 而以中心平滑所得速度随机误差最小. 另外, 多项式拟合的阶数越高, 随机误差的影响越大. 因此拟合的阶数不宜太高, 故在实际应用中, 二阶中心平滑备受人们青睐. 但对于变化剧烈的弹道段, 多项式滤波的截断误差较大, 使用二阶多项式滤波(中心平滑)的总误差(随机误差和截断误差)会增大. 文[2]考虑了固定节点的样条滤波方法. 在事后数据处理中, 文[3~5]利用自由节点样条表示理论, 系统地讨论了航天测量数据处理中建模和数据融合问题, 显著改善了弹道参数的估计精度, 并能给出测量设备的系统误差估计. 受此启发, 本文考虑用二阶样条多项式表示弹道对数据进行滤波, 从而既保持对随机误差的抑制又减小截断误差.

第2节介绍了多项式最小二乘滤波器的一些主要结论.

第3节给出了样条多项式最小二乘滤波算法. 第4节给出了仿真结果、实际应用分析和结论.

2 多项式最小二乘滤波

记飞行器位置和速度测量数据分别为 $\{x(n)\}_{n=-N}^N, \{ \dot{x}(n) \}_{n=-N}^N$. 一般假设测量数据(包括位置和速度数据, 这里以位置数据为例)在不长的时间内可用二次多项式表示为,

$$x(k) = a + b(kh) + c(kh)^2 + \varepsilon_k, \quad (1)$$

$\varepsilon_k \sim N(0, \sigma^2)$, 且 $\{\varepsilon_k\}$ 相互独立, $k = -N, \dots, 0, \dots, N$

其中 h 为采样间隔, 单位为秒. 用最小二乘法来估计参数, 有

$$\begin{cases} a = \frac{1}{(2N+1)q_4 - q_2^2} \left[q_4 \sum_{k=-N}^N x(k) - q_2 \sum_{k=-N}^N k^2 x(k) \right] \\ \quad = \sum_{k=-N}^N x(k) f_1(k) \\ b = \frac{1}{q_2 h \sum_{k=-N}^N kx(k) - \sum_{k=-N}^N f_2(k) x(k)} \\ \quad = \sum_{k=-N}^N f_3(k) x(k) \\ c = \frac{1}{[(2N+1)q_4 - q_2^2]h^2} \left[(2N+1) \sum_{k=-N}^N k^2 x(k) - q_2 \sum_{k=-N}^N x(k) \right] \\ \quad = \sum_{k=-N}^N f_3(k) x(k) \end{cases} \quad (2)$$

$$\text{其中: } \begin{cases} f_1(k) = \frac{q_4 - q_2 k^2}{(2N+1)q_4 - q_2^2} \\ f_2(k) = \frac{k}{q_2 h} \\ f_3(k) = \frac{(2N+1)k^2 - q_2}{[(2N+1)q_4 - q_2^2]^2} \\ q_2 = \frac{2N^3 + 3N^2 + N}{3} \\ q_4 = k^4 \end{cases}$$

且滤波后数据为

$$\hat{x}(k) = a + b(kh) + c(kh)^2 \\ = \sum_{k=-N}^N \left[f_1(l) + f_2(l)kh + f_3(l)(kh)^2 \right] x(l) \quad (3)$$

其滤波效率用方差比(滤波后输出数据与输入观测数据的随机误差方差之比)描述,

$$\mu^2(N, k) = \frac{\sum_{k=-N}^N \hat{x}(k)^2}{2} = \sum_{k=-N}^N \left[f_1(l) + f_2(l)kh + f_3(l)(kh)^2 \right]^2 \quad (4)$$

式(2)~(4)即给出了多项式平滑、滤波和外推的公式。当 $-N \leq k < N$ 时,上述过程为平滑,特别地 $k=0$ 为中心平滑, $k=N$ 时为滤波, $k > N$ 时为外推。

3 样条多项式最小二乘滤波模型及算法

3.1 固定节点的样条多项式最小二乘滤波

以 $x(t) = a + bt + ct^2 + dt^2_+$ 为模型,其中 $t^2_+ = \begin{cases} t^2, & t > 0 \\ 0, & t = 0 \end{cases}$, 测量数据可表示为

$$x(k) = a + b(kh) + c(kh)^2 + d(kh)^2_+ + \varepsilon_k \quad (5)$$

$\varepsilon_k \sim N(0, \sigma^2)$, 且 $\{\varepsilon_k\}$ 相互独立, $k = -N, \dots, 0, \dots, N$

其中 h 与式(1)相同。构造目标函数 $s(a, b, c, d) = \sum_{k=-N}^N [x(k) - (a + b(kh) + c(kh)^2 + d(kh)^2_+)]^2$, 使其最小可得

$$\begin{cases} a(2N+1) + ch^2 q_2 + dh^2 \frac{q_2}{2} = \sum_{k=-N}^N x(k) \\ bhq_2 + dh^2 q_3 = \sum_{k=-N}^N kx(k) \\ aq_2 + ch^2 q_4 + dh^2 \frac{q_4}{2} = \sum_{k=-N}^N k^2 x(k) \\ aq_2 + 2bhq_3 + (c+d)h^2 q_4 = 2 \sum_{k=1}^N k^2 x(k) \end{cases}$$

其中 $q_3 = \sum_{k=1}^N k^3$ 。由此

$$\begin{cases} a = \frac{\sum_{k=-N}^N [(4q_3^2 - q_2 q_4) (q_4 - k^2 q_2) - 4kq_2 q_3 q_4] x(k)}{[(2N+1)q_4 - q_2^2] (4q_3^2 - q_2 q_4)} \\ b = \frac{\sum_{k=-N}^N (2q_3(2k^2 - k^2) - kq_4) x(k)}{(4q_3^2 - q_2 q_4) h} \end{cases} \quad (6a)$$

$$\begin{cases} c = \frac{\sum_{k=-N}^N [(4q_3^2 - q_2 q_4) ((2N+1)k^2 - [q_2^2 - (2N+1)q_4](4q_3^2 - q_2 q_4)) + (2kq_3 - q_2(2k^2 - k^2))(q_2^2 - (2N+1)q_4)] x(k)}{[q_2^2 - (2N+1)q_4](4q_3^2 - q_2 q_4)} \\ d = \frac{\sum_{k=-N}^N [4kq_3 - 2(2k^2 - k^2)q_2] x(k)}{(4q_3^2 - q_2 q_4) h^2} \end{cases} \quad (6b)$$

记

$$\begin{cases} g_1(k) = \frac{(4q_3^2 - q_2 q_4)(q_4 - k^2 q_2) - 4kq_2 q_3 q_4}{[(2N+1)q_4 - q_2^2] (4q_3^2 - q_2 q_4)} \\ g_2(k) = \frac{(2q_3(2k^2 - k^2) - kq_4)}{(4q_3^2 - q_2 q_4) h} \\ g_3(k) = (4q_3^2 - q_2 q_4) \cdot \frac{((2N+1)k^2 + (2kq_3 - q_2(2k^2 - k^2))(q_2^2 - (2N+1)q_4))}{[q_2^2 - (2N+1)q_4](4q_3^2 - q_2 q_4)} \\ g_4(k) = \frac{4kq_3 - 2(2k^2 - k^2)q_2}{(4q_3^2 - q_2 q_4) h^2} \end{cases}$$

则滤波后数据为

$$\hat{x}(k) = a + b(kh) + c(kh)^2 + d(kh)^2_+ \\ = \sum_{l=-N}^N (g_1(l) + g_2(l)kh + g_3(l)(kh)^2 + g_4(l)(kh)^2_+) x(l) \quad (7)$$

其滤波效率用方差比(滤波后输出数据与输入观测数据的随机误差方差之比)描述,

$$\mu^2(N, k) = \frac{\sum_{k=-N}^N \hat{x}(k)^2}{2} = \sum_{l=-N}^N (g_1(l) + g_2(l)kh + g_3(l)(kh)^2 + g_4(l)(kh)^2_+)^2 \quad (8)$$

式(6)~(8)给出了固定节点样条多项式平滑、滤波和外推的公式。当 $-N \leq k < N$ 时,上述过程为平滑,特别地 $k=0$ 为中心平滑, $k=N$ 时为滤波, $k > N$ 时为外推。

3.2 自由节点的样条多项式最小二乘滤波

以 $x(t) = a + bt + ct^2 + d(t - T)^2_+$ 为模型,其中 $T = k_0 h$ 为节点位置,则测量数据可表示为

$$x(k) = a + b(kh) + c(kh)^2 + d(kh - k_0 h)^2_+ + \varepsilon_k \quad (9)$$

$\varepsilon_k \sim N(0, \sigma^2)$, 且 $\{\varepsilon_k\}$ 相互独立, $k = -N, \dots, 0, \dots, N$

h 与式(1)相同。目标函数 $s(a, b, c, d, k_0) = \sum_{k=-N}^N [x(kh) - (a + b(kh) + c(kh)^2 + d(kh - k_0 h)^2_+)]^2$, 可得

$$\begin{cases} a(2N+1) + ch^2 q_2 + dh^2 \sum_{k=-N}^N (k - k_0)^2_+ = \sum_{k=-N}^N x(k) \\ bhq_2 + dh^2 \sum_{k=-N}^N k(k - k_0)^2_+ = \sum_{k=-N}^N kx(k) \\ aq_2 + ch^2 q_4 + dh^2 \sum_{k=-N}^N k^2(k - k_0)^2_+ = \sum_{k=-N}^N k^2 x(k) \\ a \sum_{k=-N}^N (k - k_0)^2_+ + bh \sum_{k=-N}^N k(k - k_0)^2_+ + ch^2 \sum_{k=-N}^N k^2(k - k_0)^2_+ \end{cases}$$

$$\begin{cases} + dh^2 \sum_{k=-N}^N (k-k_0)_+^4 = \sum_{k=-N}^N (k-k_0)_+^2 x(k) \\ a \sum_{k=-N}^N (k-k_0)_+ + bh \sum_{k=-N}^N k(k-k_0)_+ + ch^2 \sum_{k=-N}^N k^2(k-k_0)_+ \\ + dh^2 \sum_{k=-N}^N (k-k_0)_+^3 = \sum_{k=-N}^N (k-k_0)_+ x(k) \end{cases} \quad (10)$$

上述方程组是关于 a, b, c, d 和 k_0 的非线性方程组,须采用迭代法求解,考虑到 N 一般不大,且 $0 \leq k_0 < N$,这里给出一种直接搜索法,即先搜索求解 k_0 ,再按线性方程组计算其它参数.

$$\begin{aligned} \text{记 } I_0(x) &= \sum_{k=-N}^N x(k), I_1(x) = \sum_{k=-N}^N kx(k), I_2(x) = \sum_{k=-N}^N k^2 x(k), J_1(x, k_0) = \sum_{k=-N}^N (k-k_0)_+ x(k), J_2(x, k_0) = \sum_{k=-N}^N (k-k_0)_+^2 x(k), \\ \text{由方程组(10)的前3个方程,可得:} \\ a &= \frac{h^2}{q_2^2 - (2N+1)q_4} \left[q_4 \sum_{k=-N}^N (k-k_0)_+^2 - q_2 \sum_{k=-N}^N k^2(k-k_0)_+^2 \right] d \\ &+ \frac{1}{q_2^2 - (2N+1)q_4} [q_2 I_1(x) - q_4 I_0(x)] \triangleq F_1 d + F_2 \\ b &= -\frac{h}{q_2} \sum_{k=-N}^N k(k-k_0)_+^2 d + \frac{I_1(x)}{hq_2} \triangleq F_3 d + F_4 \\ c &= \frac{h^2}{q_2^2 - (2N+1)q_4} \left[(2N+1) \sum_{k=-N}^N k^2(k-k_0)_+^2 - q_2 \sum_{k=-N}^N (k-k_0)_+^2 \right] d \\ &+ \frac{1}{[q_2^2 - (2N+1)q_4]h^2} [q_2 I_0(x) - (2N+1)I_2(x)] \triangleq F_5 d + F_6 \end{aligned} \quad (11)$$

把 a, b, c 的表达式代入方程组(10)的第4、5个方程,得

$$\begin{aligned} &\left[F_1 \sum_{k=-N}^N (k-k_0)_+^2 + F_3 h \sum_{k=-N}^N k(k-k_0)_+^2 + F_5 h^2 \sum_{k=-N}^N k^2(k-k_0)_+^2 \right. \\ &+ \left. h^2 \sum_{k=-N}^N (k-k_0)_+^4 \right] d + F_2 \sum_{k=-N}^N (k-k_0)_+^2 + F_4 h \sum_{k=-N}^N k(k-k_0)_+^2 \\ &+ F_6 h^2 \sum_{k=-N}^N k^2(k-k_0)_+^2 - J_2(x, k_0) = 0 \\ &\left[F_1 \sum_{k=-N}^N (k-k_0)_+ + F_3 h \sum_{k=-N}^N k(k-k_0)_+ + F_5 h^2 \sum_{k=-N}^N k^2(k-k_0)_+ \right. \\ &+ \left. h^2 \sum_{k=-N}^N (k-k_0)_+^3 \right] d + F_2 \sum_{k=-N}^N (k-k_0)_+ + F_4 h \sum_{k=-N}^N k(k-k_0)_+ \end{aligned}$$

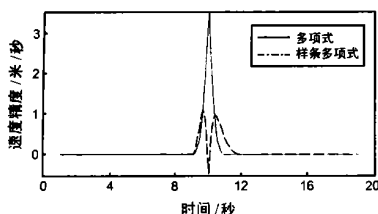


图1 二次多项式和二次样条多项式中心微分平滑截断误差图

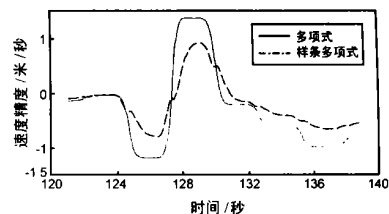
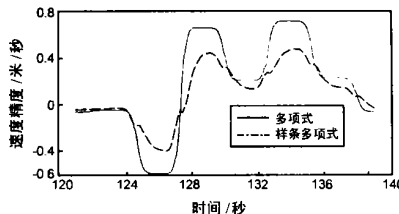


图2 二次多项式和二次样条多项式中心微分平滑截断误差图 (左图为 x 方向,右图为 y 方向)

$$+ F_6 h^2 \sum_{k=-N}^N k^2(k-k_0)_+^2 - J_1(x, k_0) = 0 \quad (12)$$

将上式记为

$$G_1 d + G_2 = 0$$

$$G_3 d + G_4 = 0$$

由此两式得,

$$G_2 G_3 - G_1 G_4 = 0$$

令 $g(k_0) = G_2 G_3 - G_1 G_4$,用逐点搜寻法求 $|g(k_0)|$ 的最小值点, k_0 的范围是 $[0, N-1]$. 得到 k_0 后,即可解线性方程组得到 a, b, c, d ,进而得到 $x(0)$ 和 $\dot{x}(0)$.

4 仿真结果与结论

例1 用 $y(t) = 5 + 6t + 7t^2 + 9(t-10)_+^2$ 产生采样间隔为 0.05 秒的共 20 秒数据,分别利用 21 点的二次多项式最小二乘滤波和二次样条多项式最小二乘滤波进行中心微分平滑,给出均方误差(与理论弹道差的均方差)和截断误差图分别见表 1 和图 1. 从图 1 和表 1 可以知道样条多项式中心微分平滑显著降低了截断误差.

例2 对某采样间隔为 0.05 秒的理论弹道分别利用 21 点的二次多项式最小二乘滤波和二次样条多项式最小二乘滤波进行中心微分平滑,给出均方误差(与理论弹道差的均方差)和截断误差图,分别见表 2 和图 2. 从图 2 和表 2 可以知道样条多项式中心微分平滑对于实际弹道数据也确实显著降低了截断误差.

表2 实测数据微分平滑均方误差

	方 向	方 向
二次多项式	0.4284 米/秒	0.7873 米/秒
样条多项式	0.2651 米/秒	0.4766 米/秒

用自由节点样条函数设计的最小二乘滤波器,与固定窗长及可变动窗长的多项式滤波器相比,既能有效地抑制随机误差,又能显著减少截断误差,仿真计算结果表明该方法能显著改善目标机动时的运动参数估计精度. 自由节点样条函数在 Kalman 类滤波器设计中的应用还有待研究.

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For assisting organization of the conference, participants are welcome to submit the Intend to Participate form to Prof. Fujiang Liao, conference chairman, by E-mail: bverizw@public3.bta.net.cn. Fax or normal mail **by the end of February, 2004**. Participants that require an invitation letter before notification of paper acceptance, please make statement at the form. Chinese participants may also send the form to Dr. Li Ji (李季).