

# 非线性协调控制系统状态方程的级数解

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**摘 要:** 针对非线性系统, 为了合理利用变量间的有益耦合、消除有害的关联, 提出了具有扰动补偿的非线性协调控制原则. 根据控制原则, 建立了非线性协调控制系统的状态方程, 基于该方程导出了对于外界扰动的完全补偿条件, 进而给出了完全补偿协调控制系统非线性状态方程. 采用直接试探法, 求得了该方程的任意阶级数解析解, 这是一种有效的非线性系统状态方程的近似求解方法.

**关键词:** 非线性协调控制; 状态方程; 级数解

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## Series Solution of the State Equation for Nonlinear Harmonic Control Systems

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**Abstract:** To a nonlinear control system, in order to utilize the useful couple between variables, and remove the harmful connection, the rules of harmonic control for nonlinear systems with disturbance compensation are given. According to the control rules, the state equation of harmonic control for nonlinear systems is established. Based on the equation, the full compensation condition eliminating extraneous disturbance, is given; and the state equation of nonlinear harmonic control systems, with full disturbance compensation, is obtained. By utilizing direct heuristic method, the any order approximate series solution of the equation can be obtained, this is a kind of useful approximate solving method of state equation for nonlinear systems.

**Key words:** nonlinear harmonic control; state equation; series solution

### 1 引言

在控制工程领域中, 许多场合要求保持若干变量之间服从某种函数关系. 各变量之间的这种函数关系称之为“协调关系”<sup>[1]</sup>, 保持各变量之间协调关系的系统为多变量“协调控制系统”. 协调控制系统中各变量之间的关系一般是非线性的, 这就相应地要求“协调控制理论”向“非线性协调控制理论”方向发展<sup>[1~5]</sup>. 对“非线性协调控制系统”, 我们已进行了初步探讨, 应用逐次迭代法, 给出了状态方程的任意阶近似解<sup>[2]</sup>. 本文在上述工作的基础上提出了“协调控制原则”, 给出了非线性协调控制系统的状态方程, 利用试探法, 求得完全补偿协调控制系统非线性状态方程的任意阶近似解.

### 2 非线性协调控制原则

对于非线性系统, 非线性协调控制原则如下<sup>[2]</sup>:

(1) 按“协调偏差”控制. 被控制系统的理想工作状

态, 不是个别被控制量的一组给定的确定值, 而是各量之间的协调关系:

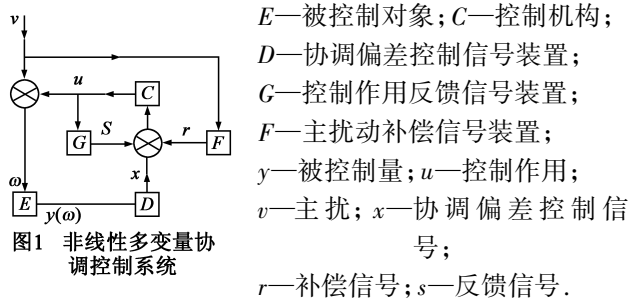
$$f(y_1, y_2, \dots, y_N) = 0 \quad (1)$$

因为系统的实际工作点相对于由协调关系所确定的协调工作点总是有所偏离的, 所以系统控制的任務就是要消除实际工作点相对于协调工作点的偏差. 为此, 可以在协调工作道上选择一适当的工作点  $\bar{P}(\bar{y}_1, \bar{y}_2, \dots, \bar{y}_N)$ , 按点  $\bar{P}$  与实际工作点  $P(y_1, y_2, \dots, y_N)$  (点  $P$  不在协调工作道上, 但对于一个可稳定工作的系统来说, 一般在协调工作道附近) 之间的状态空间坐标差进行控制, 即“协调偏差”

$$\epsilon_i(t) = \bar{y}_i - y_i(t), \quad i = 1, 2, \dots, N \quad (2)$$

(2) 采用复合控制. 对于主要干扰可测的系统, 采用按主扰控制的开环部分补偿主扰对系统协调工作的有害影响, 并与按协调偏差闭环部分相结合构成复合控制系统, 如果控制系统能完全补偿干扰的影响, 将使问题明显简化.

根据以上原则,时变非线性多变量协调控制系统框图如图 1 所示。



依照图 1 可以得到如下方程组:

$$\begin{cases} y = f(\omega) \\ \omega = u + v \\ u = f_c(x + r + s) = C(x + r + s) \\ x = f_D(\varepsilon) = D\varepsilon \\ \varepsilon = \bar{y} - y \\ r = f_F(v) = Fv \\ s = f_G(u) = Gu \end{cases} \quad (3)$$

式中,  $y, \omega, u, \gamma, x, \varepsilon, r, s$ — $N$  维列阵;  $C, D, F, G$ — $n$  阶线性算子方阵;  $y = f(\omega)$  为非线性函数。

### 3 多变量非线性协调控制系统状态方程

根据以上所述协调控制原则,系统在任意  $t$  时刻的实际工作状态,可用如下非线性常微分方程所描述:

$$\frac{dy^i(t)}{dt} = f^i(y(t), \omega(t), t) \quad (4)$$

其中,  $N$  为系统状态空间维数,时间  $t$  为自变量,  $y(t) = (y^1(t), y^2(t), \dots, y^N(t))^T$  为系统状态空间坐标,  $\omega(t) = (\omega^1(t), \omega^2(t), \dots, \omega^N(t))^T$  为控制作用  $u$  与扰动量  $v$  的和向量。

设式(4)所对应的协调工作点的解  $\bar{y}(t)$  为已知,即

$$\begin{cases} \frac{d\bar{y}^i(t)}{dt} = f^i(\bar{y}(t), \omega(t), t) \\ F(\bar{y}(t), t) = 0 \end{cases} \quad (5)$$

式(6)为协调工作道方程。其中,  $\bar{y}(t) = (\bar{y}^1(t), \bar{y}^2(t), \dots, \bar{y}^N(t))^T$

式(5)减式(4),则有:

$$\frac{d\varepsilon^i(t)}{dt} = f^i(\varepsilon(t), \omega(t), t) \quad (7)$$

其中,  $\varepsilon^i(t) = \bar{y}^i(t) - y^i(t)$ ——协调偏差,  $\varepsilon(t) = (\varepsilon^1(t), \varepsilon^2(t), \dots, \varepsilon^N(t))^T$ 。

式(7)为一般的协调偏差非线性微分方程,也即协调控制非线性状态方程的一般形式。

下面我们依据式(3)求完全补偿条件,由于式(3)在控制回路线性近似下,有:  $u(t) = C(x + r + s) = C[D\varepsilon(t) + Fv(t) + Gu(t)]$ ,经整理为:

$$u(t) = (I - CG)^{-1}CD\varepsilon(t) + (I - CG)^{-1}CFv(t)$$

$$\text{故 } \omega(t) = u(t) + v(t) = (I - CG)^{-1}CD\varepsilon(t) + [(I + (I - CG)^{-1}CF)]v(t) \quad (8)$$

假设开环反馈网络满足条件:

$$I + (I - CG)^{-1}CF = 0 \quad (9)$$

$$\text{则有: } \omega(t) = (1 - CG)^{-1}CD\varepsilon(t) \quad (10)$$

此式表明,当开环反馈网络满足式(9)时,控制作用量只取决于控制系统和协调偏差。这时扰动的作用得到了完全补偿,故称式(9)为完全补偿条件。

在完全补偿下,协调控制状态方程简化为如下形式:

$$\begin{cases} \frac{d\varepsilon^i(t)}{dt} = f^i(\varepsilon(t); (1 - CG)^{-1}CD\varepsilon(t); t) \\ \varepsilon^i(0), i = 1, 2, \dots, N \end{cases} \quad (11)$$

因为  $\varepsilon^i(t)$  是一级小量,所以式(11)可以展开为如下级数形式:

$$\begin{aligned} \frac{d\varepsilon^i(t)}{dt} = & a_{j_1}^i(t)\varepsilon^{j_1}(t) + a_{j_1 j_2}^i(t)\varepsilon^{j_1}(t)\varepsilon^{j_2}(t) \\ & + a_{j_1 j_2 j_3}^i(t)\varepsilon^{j_1}(t)\varepsilon^{j_2}(t)\varepsilon^{j_3}(t) + \dots \\ & + a_{j_1 j_2 \dots j_n}^i(t)\varepsilon^{j_1}(t)\varepsilon^{j_2}(t)\dots\varepsilon^{j_n}(t) + \dots \end{aligned} \quad (12)$$

式中的  $a$  系数取决于具体的受控对象和控制回路。

### 4 齐次方程的严格解析解

由式(12)可知,齐次状态方程为:

$$\frac{d\varepsilon^i(t)}{dt} = a_{j_1}^i(t)\varepsilon^{j_1}(t), i = 1, 2, \dots, N \quad (13)$$

改写为向量形式为:

$$\frac{d\varepsilon(t)}{dt} = A(t)\varepsilon(t) \quad (14)$$

利用 Picard 递归积分法求解式(14),即得齐次方程的严格解为<sup>[6]</sup>:

$$\begin{aligned} \varepsilon(t) = & R(t)\varepsilon(0) \\ = & [I + \int_0^t A(t_1)dt_1 + \int_0^t \int_0^{t_1} A(t_1)A(t_2)dt_2dt_1 + \dots \\ & + \int_0^t \int_0^{t_1} \dots \int_0^{t_{n-1}} A(t_1)A(t_2)\dots A(t_n) \\ & dt_n dt_{n-1} \dots dt_2 dt_1 + \dots] \varepsilon(0) \end{aligned} \quad (15)$$

其中:

$$\begin{aligned} R(t) = & I + \int_0^t A(t_1)dt_1 + \int_0^t \int_0^{t_1} A(t_1)A(t_2)dt_2dt_1 + \dots \\ & + \int_0^t \int_0^{t_1} \dots \int_0^{t_{n-1}} A(t_1)A(t_2)\dots A(t_n) \\ & dt_n dt_{n-1} \dots dt_2 dt_1 + \dots \end{aligned} \quad (16)$$

### 5 非线性微分状态方程的直接试探解法

对于非线性状态方程,无论是微分形式还是积分形式,均可采用级数直接试探法求解。

设非线性微分状态方程的解具有如下无穷级数形

式<sup>[7,8]</sup>:

$$\begin{aligned} \varepsilon^i(t) = & R_l^i(t) [\delta_{q_1}^l \varepsilon^{q_1}(0) + \beta_{q_1 q_2}^l(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \\ & + \beta_{q_1 q_2 q_3}^l(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \varepsilon^{q_3}(0) + \cdots \\ & + \beta_{q_1 q_2 \cdots q_n}^l(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots] \quad (17) \end{aligned}$$

其中,  $\delta_l^i$  为 Kronecher 符号,  $\beta$  系数为试探待定系数,  $R_l^i$  的初始条件为  $R_l^i(0) = \delta_l^i$ , 所有  $\beta$  系数的初始条件均为零.

首先将试探解式(17)代入式(12)的左端, 即对式(17)的两边对  $t$  求导, 整理得:

$$\begin{aligned} \frac{d\varepsilon^i(t)}{dt} = & a_{j_1}^i(t) \varepsilon^{j_1}(t) + R_l^i(t) \left[ \frac{d\beta_{q_1 q_2}^l(t)}{dt} \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \right. \\ & + \frac{d\beta_{q_1 q_2 q_3}^l(t)}{dt} \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \varepsilon^{q_3}(0) + \cdots \\ & \left. + \frac{d\beta_{q_1 q_2 \cdots q_n}^l(t)}{dt} \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \right] \quad (18) \end{aligned}$$

另一方面, 若令

$$T_{q_1 q_2 \cdots q_n}^i(t) = R_l^i(t) \beta_{q_1 q_2 \cdots q_n}^l(t), i = 1, 2, \cdots, N \quad (19)$$

则由式(17)可得:

$$\begin{aligned} \varepsilon^{j_1}(t) \varepsilon^{j_2}(t) = & \alpha_{q_1 q_2}^{j_1 j_2}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) + \alpha_{q_1 q_2 q_3}^{j_1 j_2}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \varepsilon^{q_3}(0) + \cdots \\ & + \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \quad (20) \end{aligned}$$

$$\begin{aligned} \varepsilon^{j_1}(t) \varepsilon^{j_2}(t) \varepsilon^{j_3}(t) = & \alpha_{q_1 q_2 q_3}^{j_1 j_2 j_3}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \varepsilon^{q_3}(0) + \cdots \\ & + \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2 j_3}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \quad (21) \end{aligned}$$

$$\begin{aligned} \varepsilon^{j_1}(t) \varepsilon^{j_2}(t) \cdots \varepsilon^{j_{k-1}}(t) = & \alpha_{q_1 q_2 \cdots q_{n-1}}^{j_1 j_2 \cdots j_{k-1}}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_{n-1}}(0) \\ & + \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2 \cdots j_{k-1}}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \quad (22) \end{aligned}$$

$$\begin{aligned} \varepsilon^{j_1}(t) \varepsilon^{j_2}(t) \cdots \varepsilon^{j_k}(t) = & \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2 \cdots j_k}(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \quad (23) \end{aligned}$$

其中,

$$\begin{cases} \alpha_{q_1 q_2}^{j_1 j_2}(t) = R_{q_1}^{j_1}(t) R_{q_2}^{j_2}(t) \\ \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2}(t) = R_{q_1}^{j_1}(t) T_{q_2 q_3 \cdots q_n}^{j_2}(t) + T_{q_1 q_2}^{j_1}(t) T_{q_3 q_4 \cdots q_n}^{j_2}(t) \\ \quad + \cdots + T_{q_2 q_3 \cdots q_{n-2}}^{j_1}(t) T_{q_{n-1} q_n}^{j_2}(t) \\ \quad + T_{q_1 q_2 \cdots q_{n-1}}^{j_1}(t) R_{q_n}^{j_2}(t) \\ \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2 j_3}(t) = \alpha_{q_1 q_2}^{j_1 j_2}(t) T_{q_3 q_4 \cdots q_n}^{j_3}(t) + \alpha_{q_1 q_2 q_3}^{j_1 j_2}(t) T_{q_4 q_5 \cdots q_n}^{j_3}(t) \\ \quad + \cdots + \alpha_{q_1 q_2 \cdots q_{n-2}}^{j_1 j_2}(t) T_{q_{n-1} q_n}^{j_3}(t) \\ \quad + \alpha_{q_1 q_2 \cdots q_{n-1}}^{j_1 j_2}(t) R_{q_n}^{j_3}(t) \\ \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2 \cdots j_{k-1}}(t) = \alpha_{q_1 q_2 \cdots q_{n-2}}^{j_1 j_2 \cdots j_{k-2}}(t) T_{q_{n-1} q_n}^{j_{k-1}}(t) \\ \quad + \alpha_{q_1 q_2 \cdots q_{n-1}}^{j_1 j_2 \cdots j_{k-2}}(t) R_{q_n}^{j_{k-1}}(t) \\ \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2 \cdots j_k}(t) = \alpha_{q_1 q_2 \cdots q_{n-1}}^{j_1 j_2 \cdots j_{k-1}}(t) R_{q_n}^{j_k}(t) \\ \quad = R_{q_1}^{j_1}(t) R_{q_2}^{j_2}(t) \cdots R_{q_n}^{j_n}(t) \end{cases} \quad (24)$$

把式(20)~(23)代入到式(12)的右端, 经整理得:

$$\begin{aligned} \frac{d\varepsilon^i(t)}{dt} = & a_{j_1}^i \varepsilon^{j_1}(t) + A_{q_1 q_2}^i(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \\ & + A_{q_1 q_2 q_3}^i(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \varepsilon^{q_3}(0) + \cdots \\ & + A_{q_1 q_2 \cdots q_n}^i(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \quad (25) \end{aligned}$$

$$\begin{aligned} \text{其中, } A_{q_1 q_2}^i(t) = & a_{j_1 j_2}^{i j_1 j_2}(t) \alpha_{q_1 q_2}^{j_1 j_2}(t), \\ A_{q_1 q_2 q_3}^i(t) = & a_{j_1 j_2}^{i j_1 j_2}(t) \alpha_{q_1 q_2 q_3}^{j_1 j_2}(t) + a_{j_1 j_2 j_3}^{i j_1 j_2}(t) \alpha_{q_1 q_2 q_3}^{j_1 j_2}(t), \\ A_{q_1 q_2 \cdots q_n}^i(t) = & a_{j_1 j_2}^{i j_1 j_2}(t) \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2}(t) \\ & + a_{j_1 j_2 j_3}^{i j_1 j_2}(t) \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2}(t) + \cdots \\ & + a_{j_1 j_2 \cdots j_n}^{i j_1 j_2}(t) \alpha_{q_1 q_2 \cdots q_n}^{j_1 j_2}(t) \end{aligned}$$

比较式(18)和式(25), 可知  $\beta$  系数满足如下方程:

$$\begin{cases} R_l^i(t) \frac{d\beta_{q_1 q_2}^l(t)}{dt} = A_{q_1 q_2}^i(t) \\ R_l^i(t) \frac{d\beta_{q_1 q_2 q_3}^l(t)}{dt} = A_{q_1 q_2 q_3}^i(t) \\ R_l^i(t) \frac{d\beta_{q_1 q_2 \cdots q_n}^l(t)}{dt} = A_{q_1 q_2 \cdots q_n}^i(t) \end{cases} \quad (26)$$

考虑到  $(R^{-1})^j R_l^i = \delta_j^i$  以及  $\delta_j^i$  的指标置换性质, 式(26)中的各式的两边各乘以  $(R^{-1})_i^j$ , 并从 0 到  $t$  积分, 再利用张量指标符号的任意选择性, 可得  $\beta$  系数为:

$$\begin{aligned} \beta_{q_1 q_2}^l(t) = & \int_0^t (R^{-1})_{i_2}^l(\tau) A_{q_1 q_2}^{i_2}(\tau) d\tau, \\ \beta_{q_1 q_2 q_3}^l(t) = & \int_0^t (R^{-1})_{i_3}^l(\tau) A_{q_1 q_2 q_3}^{i_3}(\tau) d\tau, \\ \beta_{q_1 q_2 \cdots q_n}^l(t) = & \int_0^t (R^{-1})_n^l(\tau) A_{q_1 q_2 \cdots q_n}^{i_n}(\tau) d\tau. \end{aligned}$$

把  $\beta$  系数代入到试探解式(17)中, 则得非线性状态方程的任意阶近似解析解为:

$$\begin{aligned} \varepsilon^i(t) = & R_{q_1}^i(t) \varepsilon^{q_1}(0) + T_{q_1 q_2}^i(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) + \cdots \\ & + T_{q_1 q_2 \cdots q_n}^i(t) \varepsilon^{q_1}(0) \varepsilon^{q_2}(0) \cdots \varepsilon^{q_n}(0) + \cdots \quad (27) \end{aligned}$$

其中高次项的系数为:

$$\begin{aligned} T_{q_1 q_2}^i(t) = & R_l^i(t) \int_0^t (R^{-1})_{i_2}^l(\tau) A_{q_1 q_2}^{i_2}(\tau) d\tau, \\ T_{q_1 q_2 \cdots q_n}^i(t) = & R_l^i(t) \int_0^t (R^{-1})_n^l(\tau) A_{q_1 q_2 \cdots q_n}^{i_n}(\tau) d\tau. \end{aligned}$$

综上所述, 通过直接试探法可以得到非线性状态方程的任意阶近似解。

## 6 结论

本文给出了非线性协调控制系统的控制原则, 导出了完全消除扰动的补偿条件, 得到了非线性协调控制系统状态方程, 采用直接试探法, 给出了非线性状态方程的任意阶近似级数解. 对于非线性协调控制系统的积分形式的状态方程, 同样可以得到其级数近似解析解, 并证明解的收敛性, 由于篇幅, 这里不在赘述, 这些结果将另文发表。

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