# 基于模糊变换的模糊系统和模糊推理建模法

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**摘　要：** 首先针对双输入单输出模糊系统提出了一种模糊变换方法，指出这种模糊变换不仅与单输入和单输出模糊系统有密切的联系，而且利用这种模糊变换构造的模糊系统具有很好的泛逼近性．然后将这种模糊变换应用到模糊推理建模法中，导出了所研究的模糊系统的微分方程模型和状态空间模型，从而给出了一种新的模糊推理建模法．理论分析和仿真实验均表明：利用模糊变换构造的微分方程对所研究的模糊系统具有很好的泛逼近性．

**关键词：** 模糊控制；模糊变换；泛逼近性；HX方程；状态空间模型
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# Fuzzy System and Fuzzy Inference Modeling Method Based on Fuzzy Transformation

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**Abstract：** In this paper，we puts forward a fuzzy transformation method for double-input-single-output fuzzy system，and points out that not only this fuzzy transformation has intimate connections with single-input-single-output fuzzy systems，but also the fuzzy system constructed with this fuzzy transformation has good universal approximation. By applying the fuzzy transformation to the fuzzy reasoning modeling method，we establish a new fuzzy reasoning modeling method and derive the differential equation model and the state-space model for the system which is studied. The theoretical analysis and simulate experiment show that the differential equations based on fuzzy transformation have good universal approximation.

**Key words：** fuzzy control；fuzzy transformation；universal approximation；differential equations；state space model

## 1　引言

模糊系统的构造是模糊控制领域的热点研究问题之一．模糊系统的构造主要分为以下过程[1~6]：模糊化，模糊推理和解模糊化．其中模糊化有单点模糊化和参数单点模糊化[7]，模糊推理有CRI（Compositional Rules of Inference）推理方法[8]或三I推理方法[9]，常用的解模糊化方法有中心平均解模糊化方法，重心法解模糊化方法和最大值解模糊化方法．模糊系统的构造与模糊系统的建模有密切的联系．2002年，李洪兴提出了模糊推理建模法．该方法实际上是采用中心解模糊化方法，通过利用模糊逻辑系统的插值机理[10]将既得的模糊推理规则库变为某种变系数非线性微分方程[11]．2009年，李洪兴等利用此系统通过对时间区域划分实现了时变自由运动模型的建模[12]，同样得到了微分方程形式的数学模型．

基于重心法的双输入单输出模糊系统的构造十分复杂，需要在多个小区域上进行三重积分．为了克服这一困难，本文提出了一种模糊变换方法．该方法是利用单输入单输出重心法模糊系统引入一种函数变换，再通过这种函数变换得到双输入单输出模糊系统．我们首先证明了这种模糊系统具有很好的泛逼近性，然后将这种模糊系统应用到模糊推理建模中，导出了所研究系统的微分方程模型和状态空间模型．通过对时不变和时变的自由运动模型的应用，验证了这种建模方法的有效性．

## 2　模糊变换与模糊系统的构造

对双输入单输出的Mamdani的模糊系统，由文献[7]知，对函数$f:[a_1,b_1]\times[a_2,b_2]\to[c,d]$，当$(x,y)\in[x_i,x_{i+1}]\times[y_j,y_{j+1}]$时，采用单点模糊化，乘机推理机，中心平均解模糊化所构造的模糊系统表示为

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$$\begin{aligned}\underline{S}(x,y)=&A_i(x)B_j(y)f(x_i,y_j)\\&+A_{i+1}(x)B_j(y)f(x_{i+1},y_j)\\&+A_i(x)B_{j+1}(y)f(x_i,y_{j+1})\\&+A_{i+1}(x)B_{j+1}(y)f(x_{i+1},y_{j+1})\end{aligned}\tag{1}$$

其中 $A_i(x)$, $B_j(y)$分别是以 $x_i$, $y_j$ 为峰点的具有二相性的三角波.本文引入函数变换:

$$L_1(f)=B_j(y)f(x,y_j)+B_{j+1}(y)f(x,y_{j+1})\triangleq g(x,y)$$

$$L_2(f)=A_i(x)f(x_i,y)+A_{i+1}(x)f(x_{i+1},y)$$

则

$$\begin{aligned}L_2(g)\triangleq&\underline{S}(x,y)\\=&A_i(x)B_j(y)f(x_i,y_j)\\&+A_{i+1}(x)B_j(y)f(x_{i+1},y_j)\\&+A_i(x)B_{j+1}(y)f(x_i,y_{j+1})\\&+A_{i+1}(x)B_{j+1}(y)f(x_{i+1},y_{j+1})\end{aligned}$$

即$\underline{S}(x,y)=L_1(L_2(f))=L_2(L_1(f))$,称这样的变换为模糊变换.可以推得有如下情况:

(1)设$\{B_j^*\}$满足 $B_j^*(y)+B_{j+1}^*(y)=1$,则可用$\{B_j^*\}$代替$\{B_j\}$得到模糊系统为

$$\begin{aligned}S(x,y)=&L_1(L_2(f))=L_2(L_1(f))\\=&A_i(x)B_j^*(y)f(x_i,y_j)\\&+A_{i+1}(x)B_j^*(y)f(x_{i+1},y_j)\\&+A_i(x)B_{j+1}^*(y)f(x_i,y_{j+1})\\&+A_{i+1}(x)B_{j+1}^*(y)f(x_{i+1},y_{j+1})\end{aligned}\tag{2}$$

(2)设$\{A_i^*\}$满足 $A_i^*(x)+A_{i+1}^*(x)=1$,则可用$\{A_i^*\}$代替$\{A_i\}$得到模糊系统为

$$\begin{aligned}S(x,y)=&L_1(L_2(f))=L_2(L_1(f))\\=&A_i^*(x)B_j(y)f(x_i,y_j)\\&+A_{i+1}^*(x)B_j(y)f(x_{i+1},y_j)\\&+A_i^*(x)B_{j+1}(y)f(x_i,y_{j+1})\\&+A_{i+1}^*(x)B_{j+1}(y)f(x_{i+1},y_{j+1})\end{aligned}\tag{3}$$

(3)当$\{A_i^*\}$和$\{B_j^*\}$满足 $A_i^*(x)+A_{i+1}^*(x)=1$, $B_j^*(y)+B_{j+1}^*(y)=1$,则可用$\{A_i^*\}$代替$\{A_i\}$,$\{B_j^*\}$代替$\{B_j\}$则得到模糊系统为

$$\begin{aligned}S(x,y)=&L_1(L_2(f))=L_2(L_1(f))\\=&A_i^*(x)B_j^*(y)f(x_i,y_j)\\&+A_{i+1}^*(x)B_j^*(y)f(x_{i+1},y_j)\\&+A_i^*(x)B_{j+1}^*(y)f(x_i,y_{j+1})\\&+A_{i+1}^*(x)B_{j+1}^*(y)f(x_{i+1},y_{j+1})\end{aligned}\tag{4}$$

其中$\{A_i^*\}$和$\{B_j^*\}$可由单输入单输出重心法模糊系统得到,具体方法在文献[2,13,14]中有详细介绍.

本文设 $A_i(x)$, $B_j(y)$分别是以 $x_i$, $y_j$ 为峰点的具有二相性的三角波.由文献[13]知,当 $x\in[x_i,x_{i+1}]$时,采用单点模糊化,选择 Göguen 蕴涵的圈乘算子为模糊蕴涵算子和重心法解模糊化得到的单输入单输出模糊系统为

$$\overline{S}_G(x)=A_i^*(x)y_i+A_{i+1}^*(x)y_{i+1}$$

其中:
$$A_i^*(x)=\frac{1}{3}+\frac{1}{3}A_i(x),$$
$$A_{i+1}^*(x)=\frac{1}{3}+\frac{1}{3}A_{i+1}(x)$$

为此令
$$B_j^*(y)=\frac{1}{3}+\frac{1}{3}B_j(y),$$
$$B_{j+1}^*(y)=\frac{1}{3}+\frac{1}{3}B_{j+1}(y)$$

利用模糊变换(2)得到双输入单输出模糊系统:

$$\begin{aligned}S_G(x,y)=&A_i(x)B_j^*(y)f(x_i,y_j)\\&+A_{i+1}(x)B_j^*(y)f(x_{i+1},y_j)\\&+A_i(x)B_{j+1}^*(y)f(x_i,y_{j+1})\\&+A_{i+1}(x)B_{j+1}^*(y)f(x_{i+1},y_{j+1})\end{aligned}\tag{5}$$

模糊系统式(5)有下面的泛逼近性定理.

**定理 1**　设 $f(x,y)$具有二阶连续偏导数,则

$$\begin{aligned}\|S_G-f\|_\infty\le&\frac{1}{8}\left[\left\|\frac{\partial^2 f}{\partial x^2}\right\|_\infty h_1^2+\left\|\frac{\partial^2 f}{\partial y^2}\right\|_\infty h_2^2\right]\\&+\frac{2}{3}\left\|\frac{\partial f}{\partial x}\right\|_\infty h_1+\frac{1}{3}\left\|\frac{\partial f}{\partial y}\right\|_\infty h_2\end{aligned}\tag{6}$$

其中 $h_1=\max\limits_{1\le i\le m-1}|x_{i+1}-x_i|$, $h_2=\max\limits_{1\le j\le n-1}|y_{j+1}-y_j|$.

**证明**　由文献[1]知,

$$\|\underline{S}-f\|_\infty\le\frac{1}{8}\left[\left\|\frac{\partial^2 f}{\partial x^2}\right\|_\infty h_1^2+\left\|\frac{\partial^2 f}{\partial y^2}\right\|_\infty h_2^2\right],$$

$$\begin{aligned}&S_G(x,y)-\underline{S}(x,y)\\&=(A_i(x)B_j^*(y)-A_i(x)B_j(y))\cdot f(x_i,y_j)\\&\quad+(A_{i+1}(x)B_j^*(y)-A_{i+1}(x)B_j(y))\cdot f(x_{i+1},y_j)\\&\quad+(A_i(x)B_{j+1}^*(y)-A_i(x)B_{j+1}(y))\cdot f(x_i,y_{j+1})\\&\quad+(A_{i+1}(x)B_{j+1}^*(y)-A_{i+1}(x)B_{j+1}(y))\cdot f(x_{i+1},y_{j+1})\end{aligned}$$

由于

$$\begin{aligned}&A_i(x)B_j^*(y)-A_i(x)B_j(y)\\&\quad=-(A_{i+1}(x)B_j^*(y)-A_{i+1}(x)B_j(y))\\&\qquad-(A_i(x)B_{j+1}^*(y)-A_i(x)B_{j+1}(y))\\&\qquad-(A_{i+1}(x)B_{j+1}^*(y)-A_{i+1}(x)B_{j+1}(y))\end{aligned}$$

$$\begin{aligned}&f(x_{i+1},y_{j+1})-f(x_i,y_j)\\&\quad=(f(x_{i+1},y_{j+1})-f(x_i,y_{j+1}))\\&\qquad+(f(x_i,y_{j+1})-f(x_i,y_j))\end{aligned}$$

经计算有

$$\begin{aligned}&S_G(x,y)-\underline{S}(x,y)|\\&\le A_{i+1}(x)|B_j^*(y)-B_j(y)|\cdot\left\|\frac{\partial f}{\partial x}\right\|_\infty\cdot h_1\\&\quad+|B_{j+1}^*(y)-B_{j+1}(y)|\cdot\left\|\frac{\partial f}{\partial y}\right\|_\infty\cdot h_2\\&\quad+A_{i+1}(x)|B_{j+1}^*(y)-B_{j+1}(y)|\cdot\left\|\frac{\partial f}{\partial x}\right\|_\infty\cdot h_1,\end{aligned}$$

$$\begin{aligned}&|B_{j+1}^*(y)-B_{j+1}(y)|\\&=|B_j^*(y)-B_j(y)|\end{aligned}$$

$$= \left| \frac{1}{3} - \frac{2}{3} B_j(y) \right| \le \frac{1}{3},$$

由此推出

$$\| S_G - f \|_\infty \le \| S_G - \underline{S} \|_\infty + \| \underline{S} - f \|_\infty$$

$$\le \frac{1}{8} \left[ \left\| \frac{\partial^2 f}{\partial x^2} \right\|_\infty h_1^2 + \left\| \frac{\partial^2 f}{\partial y^2} \right\|_\infty h_2^2 \right]$$

$$+ \frac{2}{3} \left\| \frac{\partial f}{\partial x} \right\|_\infty h_1 + \frac{1}{3} \left\| \frac{\partial f}{\partial y} \right\|_\infty h_2$$

**注 1** 定理 1 的式(6)给出了由式(5)确定的模糊系统具有泛逼近性的充分条件,也就是说:只要给定了逼近误差 $\varepsilon$,通过将输入变量 $x$ 和 $y$ 的输入区间做等份划分,然后利用式(6)就可确定数据点的个数,从而使式(5)逼近函数 $f$ 到指定的精度.

**例 1** 设 $s(x,y) = 0.52 + 0.1x + 0.38y - 0.06x \cdot y$

$$X = [-1,1],\ Y = [-1,1], \varepsilon = 0.1$$

取 49 个规则(将论域 $X$ 和论域 $Y$ 都做 7 等分划分).从仿真误差图形(图 1)可以看出:由式(5)所确定的模糊系统的逼近误差不超过 0.05.注意到同一例子,参考文献[15]中用了 225 条规则.

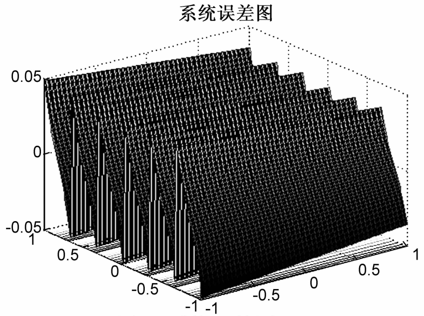

图1 系统误差图

## 3 基于模糊变换的模糊系统的建模

### 3.1 时不变系统的输入输出模型

下面讨论的系统为二阶时不变自由运动模型,设 $y(t), \dot{y}(t), \ddot{y}(t)$ 的论域分别为 $Y = [a_1, b_1]$, $\dot{y} = [a_2, b_2]$, $\ddot{Y} = [c, d]$,则 $\{y_i, \dot{y}_j, \ddot{y}_{ij}\}_{(1 \le i \le m, 1 \le j \le n)}$ 为一组输入输出数据,且满足 $a_1 = y_1 < y_2 < \cdots < y_m = b_1, a_2 = \dot{y}_1 < \dot{y}_2 < \cdots < \dot{y}_n = b_2$, $\ddot{y}_{ij}$ 在这里不作序要求.

**定理 2** 利用模糊变换可将二阶时不变自由运动模型表示为二阶变系数非线性微分方程

$$\begin{aligned} \ddot{y}(t) &= S_G(y(t), \dot{y}(t)) \\ &= b_1(y(t), \dot{y}(t)) \cdot y(t) \\ &\quad + b_2(y(t), \dot{y}(t)) \cdot \dot{y}(t) \\ &\quad + b_3(y(t), \dot{y}(t)) \cdot y(t)\dot{y}(t) \\ &\quad + b_4(y(t), \dot{y}(t)) \end{aligned} \tag{7}$$

**证明** 利用模糊变换(2),当 $(y(t), \dot{y}(t)) \in [y_i, y_{i+1}] \times [\dot{y}_j, \dot{y}_{j+1}]$ 时,$(i,j)$ 片上的局部方程为

$$\begin{aligned} \ddot{y}^{ij}(t) = {} & A_i(y(t)) \cdot \left( \frac{1}{3} + \frac{1}{3} B_j(\dot{y}(t)) \right) \cdot \ddot{y}_{ij} \\ & + A_{i+1}(y(t)) \cdot \left( \frac{1}{3} + \frac{1}{3} B_j(\dot{y}(t)) \right) \cdot \ddot{y}_{(i+1)j} \\ & + A_i(y(t)) \cdot \left( \frac{1}{3} + \frac{1}{3} B_{j+1}(\dot{y}(t)) \right) \cdot \ddot{y}_{i(j+1)} \\ & + A_{i+1}(y(t)) \cdot \left( \frac{1}{3} + \frac{1}{3} B_{j+1}(\dot{y}(t)) \right) \cdot \ddot{y}_{(i+1)(j+1)}, \end{aligned}$$

将 $A_i(y(t)), A_{i+1}(y(t)), B_j(\dot{y}(t))$ 和 $B_{j+1}(\dot{y}(t))$ 的表达式代入,则局部方程可以表示为

$$\ddot{y}^{ij}(t) = b_1^{ij} \cdot y(t) + b_2^{ij} \cdot \dot{y}(t) + b_3^{ij} \cdot y(t)\dot{y}(t) + b_4^{ij} \cdot e^{ij}$$

当 $(y(t), \dot{y}(t)) \in [y_i, y_{i+1}] \times [\dot{y}_j, \dot{y}_{j+1}]$ 时,

$$h_1 = y_{i+1} - y_i, h_2 = \dot{y}_{j+1} - \dot{y}_j$$

$$\begin{aligned} b_1^{ij} = {} & [\ddot{y}_{ij} + 2\ddot{y}_{i(j+1)} - \ddot{y}_{(i+1)j} - 2\ddot{y}_{(i+1)(j+1)}) \cdot \dot{y}_j \\ & + (\ddot{y}_{(i+1)(j+1)} + 2\ddot{y}_{(i+1)j} - \ddot{y}_{i(j+1)} - 2\ddot{y}_{ij}) \cdot \dot{y}_{j+1}] / \\ & 3h_1h_2 \end{aligned}$$

$$b_2^{ij} = [(\ddot{y}_{i(j+1)} - \ddot{y}_{ij}) \cdot y_{i+1} + (\ddot{y}_{(i+1)j} - \ddot{y}_{(i+1)(j+1)}) \cdot y_i] / 3h_1h_2,$$

$$b_3^{ij} = (\ddot{y}_{ij} - \ddot{y}_{i(j+1)} - \ddot{y}_{(i+1)j} + \ddot{y}_{(i+1)(j+1)}) / 3h_1h_2,$$

$$\begin{aligned} b_4^{ij} = {} & [(2\ddot{y}_{(i+1)(j+1)} + \ddot{y}_{(i+1)j}) \cdot y_i\dot{y}_j - (2\ddot{y}_{(i+1)j} + \\ & \ddot{y}_{(i+1)(j+1)}) \cdot y_i\dot{y}_{j+1} - (2\ddot{y}_{i(j+1)} + \ddot{y}_{ij}) \cdot y_{i+1}\dot{y}_j + (\ddot{y}_{i(j+1)} \\ & + 2\ddot{y}_{ij}) \cdot y_{i+1}\dot{y}_{j+1}] / 3h_1h_2 \end{aligned}$$

当 $(y(t), \dot{y}(t)) \notin [y_i, y_{i+1}] \times [\dot{y}_j, \dot{y}_{j+1}]$ 时,

$b_1^{ij} = b_2^{ij} = b_3^{ij} = b_4^{ij} = 0$.令

$$b_1(y(t), \dot{y}(t)) = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_1^{ij},$$

$$b_2(y(t), \dot{y}(t)) = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_2^{ij},$$

$$b_3(y(t), \dot{y}(t)) = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_3^{ij},$$

$$b_4(y(t), \dot{y}(t)) = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_4^{ij}.$$

则当 $(y(t), \dot{y}(t)) \in Y \times \dot{Y}$ 时,

$$\begin{aligned} \ddot{y}(t) &= \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \ddot{y}^{ij}(t) \\ &= \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} [b_1^{ij} \cdot y(t) + b_2^{ij} \cdot \dot{y}(t) \\ &\quad + b_3^{ij} \cdot y(t)\dot{y}(t) + b_4^{ij}] \\ &= \left( \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_1^{ij} \right) \cdot y(t) + \left( \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_2^{ij} \right) \cdot \dot{y}(t) \\ &\quad + \left( \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_3^{ij} \right) \cdot y(t)\dot{y}(t) + \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} b_4^{ij} \\ &= b_1(y(t), \dot{y}(t)) \cdot y(t) + b_2(y(t), \dot{y}(t)) \cdot \dot{y}(t) \\ &\quad + b_3(y(t), \dot{y}(t)) \cdot y(t)\dot{y}(t) + b_4(y(t), \dot{y}(t)) \end{aligned}$$

**例 2** 对 Var der Pol 方程进行仿真,该方程为 $\ddot{y}(t) - \mu(1 - y^2(t))\dot{y}(t) + y(t) = 0$,其中 $\mu = 1, y(0) = 2, \dot{y}(0) = 0$.仿真步骤参见文献[11],置 $T = 20$, $Y$ 论域上划分 12 个规则,$\dot{Y}$ 论域上划分 14 个规则,则状态曲线

$y(t)$和 $\dot{y}(t)$(实线)的仿真曲线(虚线)如图 2 ~ 图 3 所示.从图中可以看出仿真曲线与原曲线几乎重合,这说明基于模糊变换构造的微分方程模型对原系统具有较好的逼近精度.

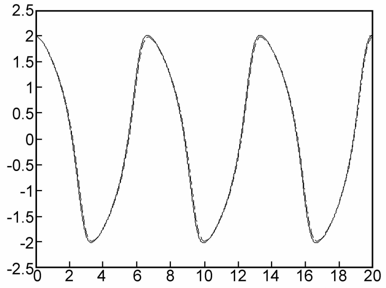

**图2　$m$=12,$n$=14时$y(t)$的仿真曲线**

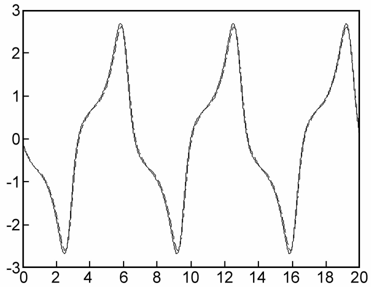

**图3　$m$=12,$n$=14时$\dot{y}(t)$的仿真曲线**

### 3.2　时不变系统的状态空间模型

仍以二阶时不变自由运动模型(即输入 $u(t)=0$)为例.设 $X_1=[a_1,b_1]$, $X_2=[a_2,b_2]$, $X_1'=[c_1,d_1]$, $X_2'=[c_2,d_2]$分别为 $x_1(t)$, $x_2(t)$, $\dot{x}_1(t)$, $\dot{x}_2(t)$的论域,模糊划分为 $A=\{A_i\}_{(1\leq i\leq m)}$, $B=\{B_j\}_{(1\leq j\leq n)}$ $C=\{C_{ij}\}_{(1\leq i\leq m,1\leq j\leq n)}$ $D=\{D_{ij}\}_{(1\leq i\leq m,1\leq j\leq n)}$. $A_i$, $B_j$, $C_{ij}$, $D_{ij}$为三角波隶属函数,它们的的峰点分别为 $x_i^{(1)}$, $x_j^{(2)}$, $\dot{x}_{ij}^{(1)}$, $\dot{x}_{ij}^{(2)}$,满足条件 $a_1=x_1^{(1)}<x_2^{(1)}<\cdots<x_m^{(1)}=b_1$, $a_2=x_1^{(2)}<x_2^{(2)}<\cdots<x_n^{(2)}=b_2$,对 $\dot{x}_{ij}^{(1)}$和$\dot{x}_{ij}^{(2)}$不作序要求.于是有模糊推理规则

$$\text{If } x_1(t) \text{ is } A_i,\ x_2(t) \text{ is } B_j,\ \text{then } \dot{x}_1(t) \text{ is } C_{ij}$$
$$\text{and } \dot{x}_2(t) \text{ is} D_{ij}(i=1,2,\cdots,m;j=1,2,\cdots,n) \quad (8)$$

则基于模糊变换建模法,该系统可以表示为一个二元分片向量拟合函数

$$(\dot{x}_1(t),\dot{x}_2(t))=(S_{G1}(x_1(t),x_2(t)),S_{G2}(x_1(t),x_2(t))),$$

$$\dot{x}_1(t)=S_{G1}(x_1(t),x_2(t))$$
$$=\sum_{i=1}^{m}\sum_{j=1}^{n}A_i(x_1(t))\cdot(\frac{1}{3}+\frac{1}{3}B_j(x_2(t)))\cdot\dot{x}_{ij}^{(1)},$$

$$\dot{x}_2(t)=S_{G2}(x_1(t),x_2(t))$$
$$=\sum_{i=1}^{m}\sum_{j=1}^{n}A_i(x_1(t))\cdot(\frac{1}{3}+\frac{1}{3}B_j(x_2(t)))\cdot\dot{x}_{ij}^{(2)}$$

**定理 3**　基于上述假定下,二阶系统的自由运动的状态空间模型可以表示为变系数非线性微分方程组:

$$\begin{aligned}\dot{x}_1(t)&=S_{G1}(x_1(t),x_2(t))\\&=b_{11}(x_1(t),x_2(t))\cdot x_1(t)+b_{12}(x_1(t),x_2(t))\cdot x_2(t)\\&\quad+b_{13}(x_1(t),x_2(t))\cdot x_1(t)x_2(t)+b_{14}(x_1(t),x_2(t))\end{aligned}\tag{9}$$

$$\begin{aligned}\dot{x}_2(t)&=S_{G2}(x_1(t),x_2(t))\\&=b_{21}(x_1(t),x_2(t))\cdot x_1(t)+b_{22}(x_1(t),x_2(t))\cdot x_2(t)\\&\quad+b_{23}(x_1(t),x_2(t))\cdot x_1(t)x_{22}(t)+b_{24}(x_1(t),x_2(t))\end{aligned}\tag{10}$$

其中,对 $k=1,2$,有

$$b_{k1}(x_1(t),x_2(t))=\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}b_{k1}^{ij},$$
$$b_{k2}(x_1(t),x_2(t))=\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}b_{k2}^{ij},$$
$$b_{k3}(x_1(t),x_2(t))=\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}b_{k3}^{ij},$$
$$b_{k4}(x_1(t),x_2(t))=\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}b_{k4}^{ij}.$$

当$(x_1(t),x_2(t))\notin[x_i^{(1)},x_{i+1}^{(1)}]\times[x_j^{(2)},x_{j+1}^{(2)}]$时,$b_{k1}^{ij}=b_{k2}^{ij}=b_{k3}^{ij}=b_{k4}^{ij}=0$.

当$(x_1(t),x_2(t))\in[x_i^{(1)},x_{i+1}^{(1)}]\times[x_j^{(2)},x_{j+1}^{(2)}]$时,$h_1=x_{i+1}^{(1)}-x_i^{(1)}$, $h_2=x_{j+1}^{(2)}-x_j^{(2)}$.

$$\begin{aligned}b_{k1}^{ij}=[&(\dot{x}_{ij}^{(k)}+2\dot{x}_{i(j+1)}^{(k)}-\dot{x}_{(i+1)j}^{(k)}-2\dot{x}_{(i+1)(j+1)}^{(k)})\cdot x_j^{(2)}\\&+(\dot{x}_{(i+1)(j+1)}^{(k)}+2\dot{x}_{(i+1)j}^{(k)}-\dot{x}_{i(j+1)}^{(k)}-2\dot{x}_{ij}^{(k)})\cdot x_{j+1}^{(2)}]/\\&3h_1h_2,\end{aligned}$$

$$\begin{aligned}b_{k2}^{ij}=[&(\dot{x}_{i(j+1)}^{(k)}-\dot{x}_{ij}^{(k)})\cdot x_{i+1}^{(1)}+(\dot{x}_{(i+1)j}^{(k)}-\dot{x}_{(i+1)(j+1)}^{(k)})\cdot x_i^{(1)}]/\\&3h_1h_2,\end{aligned}$$

$$b_{k3}^{ij}=(\dot{x}_{ij}^{(k)}-\dot{x}_{i(j+1)}^{(k)}-\dot{x}_{(i+1)j}^{(k)}+\dot{x}_{(i+1)(j+1)}^{(k)})/3h_1h_2,$$

$$\begin{aligned}b_{k4}^{ij}=[&(2\dot{x}_{(i+1)(j+1)}^{(k)}+\dot{x}_{(i+1)j}^{(k)})\cdot x_i^{(1)}x_j^{(2)}\\&-(2\dot{x}_{(i+1)(j+1)}^{(k)}+\dot{x}_{(i+1)(j+1)}^{(k)})\cdot x_i^{(1)}x_{j+1}^{(2)}\\&-(2\dot{x}_{i(j+1)}^{(k)}+\dot{x}_{ij}^{(k)})\cdot x_{i+1}^{(1)}x_j^{(2)}\\&+(\dot{x}_{i(j+1)}^{(k)}+2\dot{x}_{ij}^{(k)})\cdot x_{i+1}^{(1)}x_{j+1}^{(2)}]/3h_1h_2\end{aligned}$$

证明类似于定理 2.可见,如果求解方程组(9)(10),只需逐片求解即可.

**例 3**　在例 2 中,令 $x_1(t)=y(t)$, $x_2(t)=\dot{y}(t)$,则状态曲线 $x_1(t)$和 $x_2(t)$(实线)的仿真曲线(虚线)见图 2 ~ 图 3.

### 3.3　时变系统的输入输出模型

以双输入单输出时变系统为例,介绍一种新的时变系统建模法.

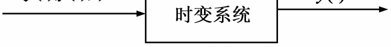

**图4　二阶时变系统**

取足够大的 $T>0$,在有限时间区间$[0,T]$上做等

距划分，$0=t_0<t_1<\cdots<t_N=T$，其中 $t_k=\frac{kT}{N}(k=0,1,\cdots,N-1)$，在每个时间间隔$[t_k,t_{k+1}](k=0,1,\cdots,N-1)$上用前面建立的时不变系统代替时变系统.在每个时间间隔内，输入变量 $y(t)$，$\dot{y}(t)$和输出变量 $\ddot{y}(t)$的论域都依赖于指标 $k$，他们分别为 $Y_k=[a_{1k},b_{1k}]$，$\dot{Y}_k=[a_{2k},b_{2k}]$，$\ddot{Y}_k=[c_k,d_k]$.对于每个指标 $k(k=0,1,\cdots,N-1)$，论域 $Y_k$，$\dot{Y}_k$ 和 $\ddot{Y}_k$ 的模糊划分为 $A_k=\{A_{ki}\}_{(1\le i\le m_k)}$，$B_k=\{B_{kj}\}_{(1\le j\le n_k)}$ 和 $C_k=\{C_{kij}\}_{(1\le i\le m_k,1\le j\le n_k)}$. $y_{ki}$，$\dot{y}_{kj}$和 $\ddot{y}_{kij}$分别为 $A_{ki}$，$B_{kj}$和 $C_{kij}$的峰点，满足条件 $a_{1k}\le y_{k1}<y_{k2}<\cdots<y_{km_k}\le b_{1k}$，$a_{2k}\le\dot{y}_{k1}<\dot{y}_{k2}<\cdots<\dot{y}_{kn_k}\le b_{2k}$，这里对 $\ddot{y}_{kij}$不作序要求.有如下推理规则组

$$\begin{aligned}&\text{If } y(t) \text{ is } A_{ki} \text{ and } \dot{y}(t) \text{ is } B_{kj}\\&\text{then } \ddot{y}(t) \text{ is } C_{kij}\\&\qquad (i=1,\cdots,m_k,j=1,\cdots,n_k)\end{aligned}\tag{11}$$

其中 $A_{ki}$和 $B_{kj}$的三角隶属函数形式参见文献[11].

**定理 4** 基于模糊变换构造的系统，二阶时变自由运动模型可以表示为

$$\begin{aligned}\ddot{y}(t)&=S_G(t,y(t),\dot{y}(t))\\&=d_1(t,y(t),\dot{y}(t))\cdot y(t)\\&\quad+d_2(t,y(t),\dot{y}(t))\cdot\dot{y}(t)\\&\quad+d_3(t,y(t),\dot{y}(t))\cdot y(t)\dot{y}(t)\\&\quad+d_4(t,y(t),\dot{y}(t))\end{aligned}\tag{12}$$

**证明** 当

$(t,y(t),\dot{y}(t))\in[t_k,t_{k+1}]\times[y_i,y_{i+1}]\times[\dot{y}_j,\dot{y}_{j+1}]$时，$(k,i,j)$片上的局部方程为

$$\begin{aligned}\ddot{y}_k^{ij}(t)&=A_{ki}(y(t))\left[\frac{1}{3}+\frac{1}{3}B_{kj}(\dot{y}(t))\right]\cdot\ddot{y}_{kij}\\&\quad+A_{k(i+1)}(y(t))\left[\frac{1}{3}+\frac{1}{3}B_{kj}(\dot{y}(t))\right]\cdot\ddot{y}_{k(i+1)j}\\&\quad+A_{ki}(y(t))\left[\frac{1}{3}+\frac{1}{3}B_{k(j+1)}(\dot{y}(t))\right]\cdot\ddot{y}_{ki(j+1)}\\&\quad+A_{k(i+1)}(y(t))\left[\frac{1}{3}+\frac{1}{3}B_{k(j+1)}(\dot{y}(t))\right]\cdot\\&\quad\ddot{y}_{k(i+1)(j+1)}\end{aligned}$$

将 $A_{ki}(y(t))$，$A_{k(i+1)}(y(t))$，$B_{kj}(\dot{y}(t))$和$B_{k(j+1)}(\dot{y}(t))$的表达式代入上述方程，经计算局部方程表示为

$$\ddot{y}_k^{ij}(t)=d_{k1}^{ij}\cdot y(t)+d_{k2}^{ij}\cdot\dot{y}(t)+d_{k3}^{ij}\cdot y(t)\dot{y}(t)+d_{k4}^{ij},$$

其中，

当$(t,y(t),\dot{y}(t))\in[t_k,t_{k+1}]\times[y_i,y_{i+1}]\times[\dot{y}_j,\dot{y}_{j+1}]$时，$h_{k1}=y_{k(i+1)}-y_{ki}$，$h_{k2}=\dot{y}_{k(j+1)}-\dot{y}_{kj}$，

$$\begin{aligned}d_{k1}^{ij}&=[(\ddot{y}_{kij}+2\ddot{y}_{ki(j+1)}-\ddot{y}_{k(i+1)j}-2\ddot{y}_{k(i+1)(j+1)})\cdot\dot{y}_{kj},\\&\quad+(\ddot{y}_{k(i+1)(j+1)}+2\ddot{y}_{k(i+1)j}-\ddot{y}_{ki(j+1)}-2\ddot{y}_{kij})\cdot\dot{y}_{k(j+1)}]/\\&\quad 3h_{k1}h_{k2},\end{aligned}$$

$$d_{k2}^{ij}=[(\ddot{y}_{ki(j+1)}-\ddot{y}_{kij})\cdot y_{k(i+1)}+(\ddot{y}_{k(i+1)j}-\ddot{y}_{k(i+1)(j+1)})\cdot y_{ki}]/3h_{k1}h_{k2},$$

$$d_{k3}^{ij}=(\ddot{y}_{kij}-\ddot{y}_{ki(j+1)}-\ddot{y}_{k(i+1)j}+\ddot{y}_{k(i+1)(j+1)})/3h_{k1}h_{k2},$$

$$\begin{aligned}d_{k4}^{ij}&=[(2\ddot{y}_{k(i+1)(j+1)}+\ddot{y}_{k(i+1)j})\cdot y_{ki}\dot{y}_{kj}\\&\quad-(2\ddot{y}_{k(i+1)j}+\ddot{y}_{k(i+1)(j+1)})\cdot y_{ki}\dot{y}_{k(j+1)}\\&\quad-(2\ddot{y}_{ki(j+1)}+\ddot{y}_{kij})\cdot y_{k(i+1)}\dot{y}_{kj}\\&\quad+(\ddot{y}_{ki(j+1)}+2\ddot{y}_{kij})\cdot y_{k(i+1)}\dot{y}_{k(j+1)}]/3h_{k1}h_{k2}\end{aligned}$$

当$(t,y(t),\dot{y}(t))\notin[t_k,t_{k+1}]\times[y_i,y_{i+1}]\times[\dot{y}_j,\dot{y}_{j+1}]$时，$d_{k1}^{ij}=d_{k2}^{ij}=d_{k3}^{ij}=d_{k4}^{ij}=0$.于是令

$$d_1(t,y(t),\dot{y}(t))=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k1}^{ij},$$

$$d_2(t,y(t),\dot{y}(t))=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k2}^{ij},$$

$$d_3(t,y(t),\dot{y}(t))=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k3}^{ij},$$

$$d_4(t,y(t),\dot{y}(t))=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k4}^{ij}$$

则当$(y(t),\dot{y}(t))\in Y\times\dot{Y}$时，

$$\begin{aligned}\ddot{y}(t)&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}\ddot{y}_k^{ij}(t)\\&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}[d_{k1}^{ij}\cdot y(t)+d_{k2}^{ij}\cdot\dot{y}(t)\\&\quad+d_{k3}^{ij}\cdot y(t)\dot{y}(t)+d_{k4}^{ij}]\\&=\left(\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k1}^{ij}\right)\cdot y(t)\\&\quad+\left(\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k2}^{ij}\right)\cdot\dot{y}(t)\\&\quad+\left(\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k3}^{ij}\right)\cdot y(t)\dot{y}(t)=\\&\quad+\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{k4}^{ij}\\&=d_1(t,y(t),\dot{y}(t))\cdot y(t)\\&\quad+d_2(t,y(t),\dot{y}(t))\cdot\dot{y}(t)\\&\quad+d_3(t,y(t),\dot{y}(t))\cdot y(t)\dot{y}(t)\\&\quad+d_4(t,y(t),\dot{y}(t))\cdot\dot{y}^2(t)\end{aligned}$$

由此可见二阶非线性时变的微分方程可以变换为逐片的非线性时不变微分方程，并且每一片上的系数均为常数.

**例 4** 给定二阶非线性时变微分方程 $\ddot{y}(t)-\dot{y}(t)^2+12ty=0$，初值为$(y_0,\dot{y}_0)=(0.6,0)$.取 $T=5$，$N=8$，$m_k=6$，$n_k=8$，$k=0,1,\cdots,7$，则状态曲线 $y(t)$和 $\dot{y}(t)$(实线)的仿真曲线(虚线)如图 5 ~ 图 6 所示.

### 3.4 时变系统的状态空间模型

仍以双输入单输出的二阶时变自由运动模型为例.对时间区间$[0,T]$做等距划分，在每个时间间隔$[t_k$，

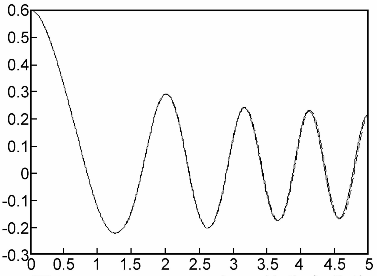

图5　$N=8,m=6,n=8$时$y(t)$的仿真曲线

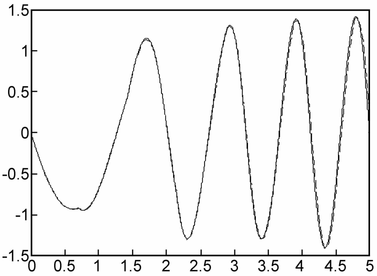

图6　$N=8,m=6,n=8$时$\dot{y}(t)$的仿真曲线

$t_{k+1}]$$(k=0,1,\cdots,N-1)$上 $x_1(t),x_2(t),\dot{x}_1(t),\dot{x}_2(t)$的论域分别为 $X_{1k}=[a_{1k},b_{1k}]$, $X_{2k}=[a_{2k},b_{2k}]$, $X'_{1k}=[c_{1k},d_{1k}]$, $X'_{2k}=[c_{2k},d_{2k}]$,论域的模糊划分分别为 $A_k=\{A_{ki}\}_{(1\le i\le m_k)}$, $B_k=\{B_{kj}\}_{(1\le j\le n_k)}$, $C_k=\{C_{kij}\}_{(1\le i\le m_k,1\le j\le n_k)}$, $D_k=\{D_{kij}\}_{(1\le i\le m_k,1\le j\le n_k)}$. $A_{ki},B_{kj},C_{kij},D_{kij}$为三角波隶属函数,它们的的峰点分别为 $x_{ki}^{(1)}$, $x_{kj}^{(2)}$, $x_{kij}^{(1)}$, $x_{kij}^{(2)}$,满足条件 $a_{k1}=x_{k1}^{(1)}<x_{k2}^{(1)}<\cdots<x_{km_k}^{(1)}=b_{k1}$, $a_{k2}=x_{k1}^{(2)}<x_{k2}^{(2)}<\cdots<x_{kn_k}^{(2)}=b_{k2}$, 对 $\dot{x}_{kij}^{(1)}$和$\dot{x}_{kij}^{(2)}$不作序要求.于是有模糊推理规则

$$\begin{aligned}&\text{If } x_1(t)\text{ is }A_{ki},x_2(t)\text{ is }B_{kj},\text{then }\dot{x}_1(t)\text{ is }C_{kij}\\&\text{and }\dot{x}_2(t)\text{ is }D_{kij}(i=1,2,\cdots,m_k;j=1,2,\cdots,n_k)\end{aligned}\tag{13}$$

则基于模糊变换建模法,该模型可以表示为一个二元分片向量拟合函数

$$\begin{aligned}(\dot{x}_1(t),\dot{x}_2(t))=(&S_{G1}(t,x_1(t),x_2(t)),\\&S_{G2}(t,x_1(t),x_2(t))),\end{aligned}$$

其中

$$\begin{aligned}\dot{x}_1(t)&=S_{G1}(t,x_1(t),x_2(t))\\&=\sum_{k=0}^{N-1}S_{Gk1}(x_1(t),x_2(t))\\&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k}\sum_{j=1}^{n_k}A_{ki}(x_1(t))\cdot[\frac{1}{3}+\frac{1}{3}B_{kj}(x_2(t))]\cdot x_{kij}^{(1)},\\\dot{x}_2(t)&=S_{G2}(t,x_1(t),x_2(t))\\&=\sum_{k=0}^{N-1}S_{Gk2}(x_1(t),x_2(t))\\&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}A_{ki}(x_1(t))\cdot[\frac{1}{3}+\frac{1}{3}B_{kj}(x_2(t))]\cdot x_{kij}^{(2)}\end{aligned}$$

**定理 5**　基于模糊变换构造的系统,二阶时变自由运动方程的状态空间模型可以表示为变系数非线性微分方程组

$$\begin{aligned}\dot{x}_1(t)&=S_{G1}(t,x_1(t),x_2(t))\\&=d_{11}(t,x_1(t),x_2(t))\cdot x_1(t)\\&\quad+d_{12}(t,x_1(t),x_2(t))\cdot x_2(t)\\&\quad+d_{13}(t,x_1(t),x_2(t))\cdot y(t)x_2(t)\\&\quad+d_{14}(t,x_1(t),x_2(t))\end{aligned}\tag{14}$$

$$\begin{aligned}\dot{x}_2(t)&=S_{G2}(t,x_1(t),x_2(t))\\&=d_{21}(t,x_1(t),x_2(t))\cdot x_1(t)\\&\quad+d_{22}(t,x_1(t),x_2(t))\cdot x_2(t)\\&\quad+d_{23}(t,x_1(t),x_2(t))\cdot y(t)x_2(t)\\&\quad+d_{24}(t,x_1(t),x_2(t))\end{aligned}\tag{15}$$

其中$(l=1,2)$,

$$\begin{aligned}d_{l1}(t,x_1(t),x_2(t))&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{kl1}^{ij},\\d_{l2}(t,x_1(t),x_2(t))&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{kl2}^{ij},\\d_{l3}(t,x_1(t),x_2(t))&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{kl3}^{ij},\\d_{l4}(t,x_1(t),x_2(t))&=\sum_{k=0}^{N-1}\sum_{i=1}^{m_k-1}\sum_{j=1}^{n_k-1}d_{kl4}^{ij}.\end{aligned}$$

当$(t,x_1(t),x_2(t))\notin[t_k,t_{k+1}]\times[x_{ki}^{(1)},x_{k(i+1)}^{(1)}]\times[x_{kj}^{(2)},x_{k(j+1)}^{(2)}]$时,$d_{kl1}^{ij}=d_{kl2}^{ij}=d_{kl3}^{ij}=d_{kl4}^{ij}=0$.

当$(t,x_1(t),x_2(t))\in[t_k,t_{k+1}]\times[x_{ki}^{(1)},x_{k(i+1)}^{(1)}]\times[x_{kj}^{(2)},x_{k(j+1)}^{(2)}]$时, $h_{k1}=x_{k(i+1)}^{(1)}-x_{ki}^{(1)}$, $h_{k2}=x_{k(j+1)}^{(2)}-x_{kj}^{(2)}$,

$$\begin{aligned}d_{kl1}^{ij}=&[(\dot{x}_{kij}^{(l)}+2\dot{x}_{ki(j+1)}^{(l)}-\dot{x}_{k(i+1)j}^{(l)}-2\dot{x}_{k(i+1)(j+1)}^{(l)})\cdot x_{kj}^{(2)},\\&+(\dot{x}_{k(i+1)(j+1)}^{(l)}+2\dot{x}_{k(i+1)j}^{(l)}-\dot{x}_{ki(j+1)}^{(l)}-2\dot{x}_{kij}^{(l)})\cdot\\&x_{k(j+1)}^{(2)}]/3h_{k1}h_{k2}\\d_{kl2}^{ij}=&[(\dot{x}_{ki(j+1)}^{(l)}-\dot{x}_{kij}^{(l)})\cdot x_{k(i+1)}^{(1)}+(\dot{x}_{k(i+1)j}^{(l)}-\\&\dot{x}_{k(i+1)(j+1)}^{(l)})\cdot x_{ki}^{(1)}]/3h_{k1}h_{k2},\\d_{kl3}^{ij}=&(\dot{x}_{kij}^{(l)}-\dot{x}_{ki(j+1)}^{(l)}-\dot{x}_{k(i+1)j}^{(l)}+\dot{x}_{k(i+1)(j+1)}^{(l)})/3h_{k1}h_{k2},\\d_{kl4}^{ij}=&[(2\dot{x}\rrbracket_{k(i+1)(j+1)}^{(l)}+\dot{x}_{k(i+1)j}^{(l)})\cdot x_{ki}^{(1)}x_{kj}^{(2)}\\&-(2\dot{x}_{k(i+1)(j+1)}^{(l)}+\dot{x}_{k(i+1)(j+1)}^{(l)})\cdot x_{ki}^{(1)}x_{k(j+1)}^{(2)}\\&-(2\dot{x}_{ki(j+1)}^{(l)}+\dot{x}_{kij}^{(l)})\cdot x_{k(i+1)}^{(1)}x_{kj}^{(2)}\\&+(\dot{x}_{ki(j+1)}^{(l)}+2\dot{x}_{kij}^{(l)})\cdot x_{k(i+1)}^{(1)}x_{k(j+1)}^{(2)}]/3h_{k1}h_{k2}\end{aligned}$$

**例 5**　在例 4 中,令 $x_1(t)=y(t)$, $x_2(t)=\dot{y}(t)$,则状态曲线 $x_1(t)$和 $x_2(t)$(实线)仿真曲线(虚线)如图 5 ~图 6 所示.

**注 2**　(1) 定理 2 ~ 定理 5 中的式(7),式(9)(10),式(12)和式(14)(15)都是利用式(5)推导出的,因此定理 1 的逼近误差估计式(6)对上述公式都适用,因此我们建立的微分方程模型均能逼近已有的微分方程到指定的精度.换句话说,我们建立的微分方程模型在理论

上保证了他们的泛逼近性.

(2)我们给出的建模方法与文献[11,12]的方法既有区别又有联系.文献[11,12]的方法是在建立推理规则后,用中心平均解模糊化方法得到模糊系统,然后将得到的模糊系统转化成微分方程模型.我们的方法是利用模糊变换直接得到模糊系统(即式(5),这样做省略了解模糊化过程),然后再将模糊系统转化为微分方程模型.本文方法与文献[11,12]的另一个区别是我们给出了定理1(即逼近误差式(6)),从而在理论上保证了所建立模型的泛逼近性.

## 4 结论

本文给出了一种新的模糊变换,这种模糊变换与单输入单输出重心法模糊系统相联系.利用这种模糊变换不仅可构造具有泛逼近性的双输入单输出模糊系统,还可利用已知数据构造系统的微分方程模型和状态空间模型.因此本文建立了一种新的模糊推理建模方法.

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